Indian J. Phys. **75B** (6), 555–563 (2001)

I J P B
— an international journal

Kinetic Alfvén wave in the presence of parallel electric field in an inhomogeneous magnetosphere

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Received 15 May 2001, accepted 11 September 2001

Abstract . Dispersion relation, associated current and growth-rate of the kinetic Alfvén wave in the presence of parallel electric field and an inhomogeneous magnetic field have been obtained by investigating the trajectories of the charged particles The effect of parallel electric field is included in the zeroth order distribution function through modification of the particle thermal velocity parallel to ambient magnetic field The plasma under consideration is assumed to be anisotropic and with low β The results are interpreted for the space plasma parameters appropriate to the auroral acceleration region of the earth's magnetoplasma

Keywords Kinetic-Alfvén wave, auroral electrodynamics, magnetosphere-ionosphere coupling

PACS Nos. 52 35 Hr, 94 10.Rk

1. Introduction

Small-scale intense disturbances of the electric field are constantly measured by polar-orbiting and Freja satellites in the altitude range from 900 km to $2R_E$ above the auroral ionosphere [1–4]. Direct measurements from satellites [5–9] and rockets [10] have shown that the discrete flux of keV electrons registered at auroral zone are often correlated with small-scale, localized electromagnetic disturbances sometimes interpreted as kinetic Alfvén waves (KAW).

It has been shown explicitly that finite-ion-gyroradius and electron-inertia effects produce parallel electric field in kinetic Alfvén waves, which brings about collisionless wave-particle resonant interaction, resulting in enhanced plasma heating and anomalous transport.

The variety of approach considers transient phenomena and explains the auroral particle acceleration in terms of linear Alfvén waves propagating parallel to or obliquely with respect to the ambient magnetic field [11–17]. These theories mainly describe the field-aligned current and parallel electric field associated with kinetic Alfvén waves which have originated in some magnetospheric generator region *via* impressed perpendicular and time varying electric fields [18]. There is observational evidence for parallel electric fields as large as 100 mV/m along auroral field lines [19,20]. Variety of theories have been proposed to explain the development of parallel electric fields on the auroral field lines [18,21], such as double layer process, electrostatic shock model and others [22]. The static model considers the d.c. electric field which is developed when two different dynamical conditions prevail between the ionosphere and magnetosphere. The purpose of this paper is to investigate the effect of such a static parallel electric field on the kinetic Alfvén wave attributed to the auroral acceleration processes in the inhomogeneous magnetosphere.

In the recent past, the method of particle aspect analysis was used by Baronia and Tiwari [17] to analyse Alfvén wave and kinetic Alfvén wave for the homogeneous magnetic field. Our purpose in this paper is to investigate the effect of parallel electric field on the kinetic Alfvén wave in the presence of an inhomogeneous magnetic field in a low β plasma using the particle aspect analysis. The basic assumptions are those of earlier work on the kinetic Alfvén wave [17] in which the plasma has been considered to consist of resonant and non-resonant particles and the

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wave growth was discussed by the energy conservation method.

We have considered a kinetic Alfvén wave propagating obliquely to the constant magnetic field, and two different potentials in the x-z plane for the evaluation of the charged particle trajectory. The direction of the density gradient and magnetic field gradient is along the y-axis.

The organization of this paper is as follows. In Section 2, we evaluate the trajectories of the charged particles and in Section 3, the density perturbation is considered. In Section 4, the dispersion relation is derived. Section 5 deals with the current densities and in Section 6, the energy balance and growth rate are estimated. Marginal stability criteria is derived in Section 7. Results and discussion are presented in Section 8.

2. Basic trajectories

In the mathematical analysis we follow the procedure considered in Refs. [17,23-26]. The kinetic Alfvén wave is assumed to start at $t = 0$ when the resonant particles are undisturbed. The main interest lies in the behaviour of those kinetic Alfvén waves which satisfy the conditions

$$V_{T\parallel i} << \frac{\omega}{k_\parallel} << V_{T\parallel e},\ \omega << \Omega_i, \Omega_e;\ k_\perp^2\rho_e^2 << k_\perp^2\rho_i^2 < 1 \quad (1)$$

where $V_{T\parallel i}$ and $V_{T\parallel e}$ are the mean velocities of ions and electrons along the magnetic field, $\Omega_{i,e}$ are gyration frequencies and $\rho_{i,e}$ the mean gyroradii of the respective species. $k_\perp$ and $k_\parallel$ are the components of real wave vector $\bar{k}$ perpendicular and parallel to the magnetic field.

We begin with the two potential representations of wave electric field of the form [17,27]

$$E_\perp = -\nabla_\perp \phi$$

and $$E_\parallel = -\nabla_\parallel \psi$$

to decouple the compressional mode and the wave electric field.

$$\bar{E} = E_\perp + E_\parallel,$$

$$\phi = \phi_1 \cos(k_\perp x + k_\parallel z - \omega t),$$

$$\psi = \psi_1 \cos(k_\perp x + k_\parallel z - \omega t), \quad (2)$$

where ϕ_1 and ψ_1 are assumed to be a slowly varying function of time t, and ω is the wave frequency which is assumed as real.

The equation of motion of particle is

$$m\frac{d\bar{v}}{dt} = q\left(\bar{E} + \frac{1}{c}\bar{v} \times \bar{B}\right), \quad (3)$$

where the collision between particles are neglected. q is charge and m is the mass of the particle and c represents the velocity of light. The velocity v can be expressed as a sum of the unperturbed velocity $\bar{V}$ and the perturbed velocity $\bar{u}$, i.e. $\bar{v} = \bar{V} + \bar{u}$, $\bar{u}$ is determined by the following sets of equations [17]

$$\frac{du_+}{dt} + i\Omega u_+ = \frac{q}{m}\left[\phi_1 k_\perp - \frac{V_\parallel k_\perp k_\parallel}{\omega}(\phi_1 - \psi_1)\right] \times \sin(k_\perp x + k_\parallel z - \omega t),$$

$$\frac{du_\parallel}{dt} = \frac{q}{m}\left[\psi_1 k_\parallel - \frac{V_\perp k_\perp k_\parallel}{\omega}(\phi_1 - \psi_1)\sin(\Omega t - \theta)\right] \times \sin(k_\perp x + k_\parallel z - \omega t), \quad (4)$$

where $u_+ = u_x + iu_y$, θ is the initial phase of velocity, $\Omega = qB_0/mc$. u_x and u_y are the perturbed velocities in the x and y directions respectively. The slowly varying quantities ϕ_1 and ψ_1 are treated as a constant. We start by taking the trajectories of free gyration as [24,25,28]

$$x(t) = x_0 + \frac{V_\perp}{\Omega}[\cos(\Omega t - \theta) + \cos\theta] + \frac{\epsilon_B V_\perp^2}{2\Omega^2} \times \left\{\Omega t + \frac{1}{2}[\sin(2\Omega t - 2\theta) + \sin 2\theta] + (\cos\Omega t - 1)\sin 2\theta - 2\sin\Omega t\cos^2\theta\right\},$$

$$y(t) = y_0 + \frac{V_\perp}{\Omega}[\sin(\Omega t - \theta) + \sin\theta] + \frac{\epsilon_B V_\perp^2}{2\Omega^2} \times \left\{\frac{1}{2}[-\cos(2\Omega t - 2\theta) + \cos 2\theta] + 2(\cos\Omega t - 1)\cos^2\theta + \sin\Omega t \sin 2\theta\right\},$$

$$z(t) = z_0 + V_\parallel t, \quad (5)$$

where $\epsilon_B = B^{-1}dB/dy$ is the inverse scale length of the magnetic field gradient, $\bar{r}_0 = (x_0, y_0, z_0)$ is the initial position of the particles at $t = 0$, where the wave is assumed to start.

Eq. (4) is solved by replacing the coordinates of charged particles to that of free gyration, which provides the perturbed velocity $\bar{u}(t)$, and can be further transformed to $\bar{u}(\bar{r}, t)$ by the use of eq. (5) once again.

Thus,

$$u_x(\bar{r}, t) = -\frac{q}{m}\left[\phi_1 k_\perp - \frac{V_\parallel k_\perp k_\parallel}{\omega}(\phi_1 - \psi_1)\right] \times A_1 A_2 \left[\frac{\Lambda_n}{a_n^2}\cos\xi_{nl} - \frac{\delta}{2\Lambda_{n+1}}\cos(\xi_{nl} - \Lambda_{n+1}t) - \frac{\delta}{2\Lambda_{n-1}}\cos(\xi_{nl} - \Lambda_{n-1}t)\right],$$

$$u_y(\bar{r}, t) = -\frac{q}{m}\left[\phi_1 k_\perp - \frac{V_\parallel k_\perp k_\parallel}{\omega}(\phi_1 - \psi_1)\right] \times A_1 A_2 \left[-\frac{\Omega}{a_n^2}\sin\xi_{nl} - \frac{\delta}{2\Lambda_{n+1}}\sin(\xi_{nl} - \Lambda_{n+1}t) + \frac{\delta}{2\Lambda_{n-1}}\sin(\xi_{nl} - \Lambda_{n-1}t)\right],$$

$$u_z(\bar{r},t) = -\frac{q}{m}\left[\psi_1 k_\perp - \frac{V_\perp k_\| k_\perp}{\omega}(\phi_1 - \psi_1)\frac{n_1}{\alpha_1}\right]A_1 A$$
$$\times \frac{1}{\Lambda_n}\left[\cos\xi_{nl} - \delta\cos(\xi_{nl} - \Lambda_n t)\right], \quad (6)$$

where $\delta = 0$ for the non-resonant particles and $\delta = 1$ for the resonant one and

$$A_1 = \sum_{n_1,n_2,\ldots n_6=-\infty}^{\infty} J_{n1}(\alpha_1)J_{n2}(\alpha_2/2)J_{n3}(\alpha_2/2)$$
$$\times J_{n4}(\alpha_2/2)J_{n5}(\alpha_2/2)J_{n6}(\alpha_2/2),$$

$$A_2 = \sum_{l_1,l_2,\ldots l_6=-} J_{l1}(\alpha_1)J_{l2}(\alpha_2/2)J_{l3}(\alpha_2/2)$$
$$\times J_{l4}(\alpha_2/2)J_{l5}(\alpha_2/2)J_{l6}(\alpha_2/2),$$

$$\alpha_1 = (k_\perp V_\perp/\Omega), \alpha_2 = (k_\perp \in_B V_\perp^2/2\Omega^2),$$

$$\Lambda_n = k_\| V_\| - \omega + \alpha_2\Omega - (n_1 - 2n_2 + n_3 + n_4 + n_5 + n_6)\Omega,$$
$$= k_\| V_\| - \omega + \alpha_2\Omega - n\Omega,$$

$$a_n^2 = \Lambda_n^2 - \Omega^2, n = (n_1 - 2n_2 + n_3 + n_4 + n_5 + n_6),$$

$$\xi_{nl} = k_\perp x + k_\| z - \omega t + \{-(n_1 - l_1) + 2(n_2 - l_2) - (n_3 - l_3)$$
$$-(n_4 - l_4) - (n_5 - l_5) - (n_6 - l_6)\}$$
$$\times \Omega t + (n_1 - l_1).(\pi/2 + \theta)$$
$$-\{(n_2 - l_2) - (n_3 - l_3) - (n_4 - l_4)\}.2\theta. \quad (7)$$

Also use was mode of the following expressions :

$$\exp[-i\mu\sin(\theta - \Omega t)] = \sum_{-\infty}^{\infty} J_n(\mu)\exp[-in(\theta - \Omega t)],$$

$$\cos\theta\exp[-i\mu\sin\theta] = \frac{n}{\mu}\sum_{-\infty}^{\infty} J_n(\mu)\exp[-in\theta].$$

Integration of eq. (6) gives the perturbed coordinates of the particles x, y, z which in addition to trajectories of free gyration exhibits the true path of the particles. In view of the approximations introduced in the beginning, the dominant contribution comes from the term $n_1 = 0$. The resonant criterion is given by $k_\| V_\| - \omega + k_\perp \in_B V_\perp^2/2\Omega^2 = 0$. The particles satisfying the above condition are called resonant. J_s are Bessel's functions which arise from the different periodical variation of charged particles trajectories. The term represented by Bessel's functions shows the reduction of the field intensities due to finite gyroradius effect.

3. Density perturbation

In order to find out the density perturbation associated with the velocity perturbation $\bar{u}(\bar{r},t,\bar{V})$, we consider the equation [17,23]

$$\frac{dn_1}{dt}(\bar{r},t,\bar{V}) = -\{\nabla.\bar{u}(\bar{r},t,\bar{V})\}$$
$$\times N - u_y(\bar{r},t,\bar{V})\frac{dN}{dy}(y,\bar{V}), \quad (8)$$

where $N(\bar{V})$ represents the zeroth-order distribution function. The eq. (8) is derived by the conservation of particle numbers [23] and perturbed quantities $n_1(r,t,V)$ and $u(r,t,V)$ depend on instantaneous velocity V. Since the density $n_1(r,t,V)$ and velocity $u(r,t,V)$ are depending on velocity the average value of density is obtained as the integrated perturbed density in eq (12). Expressing the right-hand-side of the eq. (8) as a function of t [24] and after integration, we obtain the perturbed density for non-resonant and resonant particles in the presence of the kinetic Alfvén wave for the inhomogeneous plasma as

$$n_1(\bar{r},t) = N(\bar{V})A_1A_2\frac{q}{m}\left[\left\{\phi_1 - \frac{V_\| k_\|}{\omega}(\phi_1 - \psi_1)\right.\right.$$
$$\left.\frac{k_\perp^2}{a_n^2} + \frac{\Omega^2 V_d k_\perp m}{\Lambda_n a_n^2 T_\perp}\right\} + \frac{k_\|^2}{\Lambda_n^2}$$
$$\left.\left\{\psi_1 - \frac{n_1}{\alpha_1}\frac{V_\perp k_\perp}{\omega}(\phi_1 - \psi_1)\right\}\right]\cos\xi_{nl} \quad (9)$$

and for resonant particles as

$$n_1(\bar{r},t) = N(\bar{V})A_1A_2\frac{q}{m}\left[\left\{\phi_1 - \frac{V_\| k_\|}{\omega}(\quad - \psi_1)\right.\right.$$
$$\left.\frac{k_\perp^2}{a_n^2} + \frac{\Omega^2 V_d k_\perp m}{\Lambda_n a_n^2 T_\perp}\right)\cos\xi_{nl} + \frac{1}{2\Omega\Lambda_{n+1}}$$
$$\times\cos(\xi_{nl} - \Lambda_{n+1}t)\left(k_\perp^2 - \frac{k_\perp m\Omega}{T_\perp}\right)$$
$$+ \frac{V_d k_\perp m}{\Lambda_n T_\perp}\cos(\xi_{nl} - \Lambda_n t) - \frac{1}{2\Omega\Lambda_{n-1}}$$
$$\left.\times\cos(\xi_{nl} - \Lambda_{n-1}t)\left(k_\perp^2 + \frac{V_d k_\perp m\Omega}{T_\perp}\right)\right\} + \frac{k_\|^2}{\Lambda_n^2}$$
$$\times\left\{\psi_1 - \frac{n_1}{\alpha_1}\frac{k_\perp V_\perp}{\omega}(\phi_1 - \psi_1)\right\}$$
$$\times\{\cos\xi_{nl} + \Lambda_n t\sin(\xi_{nl} - \Lambda_n t)$$
$$\left.-\cos(\xi_{nl} - \Lambda_n t)\}\right], \quad (10)$$

where V_d is the diamagnetic drift velocity which is defined by

$$V_d = \frac{T_\perp}{m\Omega}\frac{1}{N}\frac{\partial N}{\partial y} = \frac{T_\perp}{m\Omega}\in_N, \in_N = \frac{1}{N}\frac{\partial N}{\partial y}$$

and $V_d = 0$ represents the homogeneous plasma. To determine the dispersion relation and the growth rate, we use the bi-Maxwellian plasma with density distribution [24]

$$N(y,\bar{V}) = N_0\left[1 - \in\left(y + \frac{V_x}{\Omega}\right)\right]f_\perp(V_\perp)f_\|(V_\|),$$

where $f_\perp(V_\perp) = \frac{m}{2\pi T_\perp}\exp\left[-mV_\perp^2/2T_\perp\right]$, (11)

$$f_\parallel(V_\parallel) = \left(\frac{m}{2\pi T_{\parallel e}}\right)^{1/2} \exp\left[-mV_\parallel^2/2T_{\parallel e}\right],$$

$$T_{\parallel ee} = T_\parallel \left| 1 + i\frac{e\bar{E}_0 . k}{k^2 T_{\parallel e}}\right.$$

where $k = \left(k_\perp^2 + k_\parallel^2\right)^{1/2}$.

Here, $T_\perp$ and $T_\parallel$ are the perpendicular and parallel temperatures (in energy units) and $\in$ is a small parameter of the order of inverse of the density gradient scale length. The expression for $T_{\parallel ee}$ is originally derived by Pines and Schrieffer [29] adopting the rigorous treatment of kinetic approach for collective behavior of solid state plasma. They have arrived at the results where the parallel electric field E_0 is eliminated by adopting the expression for $T_{\parallel e}$ expressed as above. Their work also describes that the wave vector $\bar{k}$ at an angle to the E_0 (eq. 2.24 of Ref. [29]). They have clearly mentioned that the sign of the effect depends upon both the charge of the particle and the angle between $\bar{E}_0$ and $\bar{k}$. Thus it is true for the finite $k_\perp$ also. In this description the parallel electric field E_0 is sufficiently weak that the drift velocity of the charged particles is much smaller than the phase velocity of the wave. $e\psi/T_e < 1$ and time scales are such that the relaxations are nearly Maxwellian and the runaway conditions of the electrons are excluded by the same reasoning as discussed by Tiwari and Varma [24]. Here we follow the technique of Pines and Schrienffer [29] and Bers and Brueck [30], where a change in the zeroth-order distribution function is due to the result of change in the temperature parallel to the parallel electric field E_0. This method was further considered by Misra *et al* [31] for the investigation of whistler mode instability and Tiwari and Varma [24] for the investigation of drift instability. The existence of parallel static electric fields in the presence of KAW on auroral field lines may be the matter of further debate.

4. Dispersion relation

To evaluate the dispersion relation, we calculate the integrated perturbed density for non-resonant particles as

$$\tilde{n}_{i,e} = \int_0^\infty 2\pi V_\perp dV_\perp \int_{-\infty}^{\infty} dV_\parallel\, n_{i,e}\left(\bar{r},t,\bar{V}\right). \tag{12}$$

With the help of eqs. (9) and (11), we find the average densities for inhomogeneous plasma as

$$\tilde{n}_i = \frac{\omega_{pi}^2}{4\pi e}\left[\left(-\frac{k_\perp^2\phi}{\Omega_i^2} + \frac{k_\parallel^2\psi}{\omega^2} + \frac{V_d' k_\perp m_i}{T_{\perp i}\omega}\phi\right).\left(1 - \frac{1}{2}k_\perp^2\rho_i^2\right)\right.$$
$$\left. + \left(\frac{k_\perp^2 V_d' \in_B}{\Omega\omega^2}\phi + \frac{2T_\perp k_\perp \in_B k_\parallel^2}{m_i\Omega\omega^3}\psi\right).\left(1 - k_\perp^2\rho_i^2\right)\right], \tag{13a}$$

$$n_e - \; 4\pi e V'^2_{T\parallel ee} \quad \psi$$

Taking a complex $V'_{T\parallel ee}$ of the form

$$V'_{T\parallel ee} = V_{T\parallel e}\left(1 + ieE_0/mkV'^2_{T\parallel e}\right)^{1/2},$$

$$\tilde{n}_e = \left(\omega_{pe}^2\psi\right)/\left(4\pi e V'^2_{T\parallel e}\left[1 + \left(e^2/m_e^2\right)E_0^2/k^2 V'^4_{T\parallel e}\right]\right) \tag{13b}$$

In this model, the evaluation of dispersion relation and growth rate is based upon the real quantities and concept of imaginary quantity in various parameters has not been adopted. Therefore, in density also only real term has been considered otherwise the dispersion relation would have been complex due to this imaginary term which violates the basic principle. It is observed that essential feature of the kinetic Alfvén wave is retained even in this ideal case. For Maxwell's equation we use the quasi-neutrality condition [32]

$$\tilde{n}_i = \tilde{n}_e$$

to we get the relation between ϕ and ψ as

$$\phi = -\frac{\Omega_i^2}{k_\perp^2}\frac{\omega_{pe}^2}{\omega_{pi}V'^2_{T\parallel e}\left[1 + \left(e^2/m_e^2\right)E_0^2/k^2V'^4_{T\parallel e}\right]}$$

$$\frac{k_\parallel^2}{\omega^2}\left(1 - \frac{1}{2}k_\perp^2\rho_i^2\right). - 2\frac{\omega_{Bi}k_\parallel^2}{\omega^3}\left(1 - k_\perp^2\rho_i^2\right)\Bigg]$$

$$D_d\left[1 - \frac{1}{2}k_\perp^2\rho_i^2\right) - \frac{V_d'\Omega\in_B}{\omega^2}\left(1 - k_\perp^2\rho_i^2\right)\Big|.\psi \tag{14}$$

Using perturbed ion and electron densities $\tilde{n}_i$ and $\tilde{n}_e$ and Ampere's law in the parallel direction [32], we obtained the equation

$$\frac{\partial}{\partial z}\nabla_\perp^2(\phi - \psi) = \frac{4\pi}{c^2}\frac{\partial}{\partial t}J_z, \tag{15}$$

$$\text{where } J_z = e\int^\infty 2\pi V_\perp dV_\perp \int^\infty dV_\parallel\left[\left(N(\bar{V})u_z(\bar{r},t) + V_\parallel n_1(\bar{r},t)\right)_i\right.$$
$$\left. - \left(N(\bar{V})u_z(\bar{r},t) + V_\parallel n_1(\bar{r},t)\right)_e\right].$$

J_z is the current density which is contributed by first-order perturbations of density and velocity. With the help of eqs. (14) and (15), we obtain the dispersion relation for the kinetic Alfvén wave in inhomogeneous plasma as

$$\left(1 - \frac{\omega^2}{k_\parallel^2 c_s^2 A\left[1 + \left(e^2/m_e^2\right)E_0^2/k^2V_{T\parallel e}^4\right]}\right).\left(1 - \frac{\omega^2}{k_\parallel^2 v_A^2}D_A\right)$$
$$= \frac{k_\perp^2\omega^2}{k_\parallel^2\Omega_i^2 A}D_A - \frac{\omega_{pi}^2\omega^2 A}{c^2\Omega_i^2 k_\parallel^2}\left(\frac{T_{\parallel i}}{m_i}\right).$$

$$\left(\frac{\omega_{pe}}{\omega_{pi}^2 V_{T\|e}^2 A\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}-\frac{k_\|^2}{\omega^2}\right.$$

$$\left.-\frac{v_d' k_\perp m_i}{T_{\perp i}\omega A}D_A\right)+\frac{\omega_{pi}^2\omega_{Bi}}{c^2\Omega_i^2 k_\|^2}\left(\frac{T_{\|i}}{T_{\perp i}}\right)B\left(k_\perp v_d' D_d\right.$$

$$\cdot\omega\left(\frac{T_{\perp i}}{T_{\|i}}\right)D_d-\frac{\omega_{pe}^2 v_d'\Omega^2}{\omega_{pi}^2 k_\perp V_{T\|e}^2\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}$$

$$+\frac{v_d' k_\|^2\Omega_i^2}{\omega^2 k_\perp}+2\frac{k_\|^2 T_{\perp i}}{\omega m_i}-2\frac{AB}{\omega}\omega_{Bi}, \tag{16}$$

where $D_d=\left(1-\frac{v_d' k_\perp \Omega_i^2 m_i}{T_{\perp i}k_\perp^2\omega}\right.$

$$A=\left(1-\frac{1}{2}k_\perp^2\rho_i^2\right);\ B=\left(1-k_\perp^2\rho_i^2\right),$$

$$D_A=\left[D_d\left(1-\frac{1}{2}k_\perp^2\rho_i^2\right)-\frac{v_d'\Omega\in_B}{\omega^2}\left(1-k_\perp^2\rho_i^2\right)\right.$$

and $\omega_{Bi}=\dfrac{k_\perp \in_B T_{\perp i}}{m_i\Omega_i}$,

where $c_s^2=\dfrac{\omega_{pi}^2 V_{T\|e}^2}{\omega_{pe}}$

is the square of ion-acoustic speed and

$$v_A^2=\frac{c^2\Omega_i^2}{\omega_{pi}^2}$$

is the square of Alfvén speed. The dispersion relation of the kinetic Alfvén wave reduces to that derived by Hasegawa and Chen [17] and Baronia and Tiwari [33] under the approximation, $v_d'=0$, $\in_B=0$ and $I_0(\lambda_i)\exp(-\lambda_i)\sim 1-\lambda_i$, for $\lambda_i=\frac{1}{2}k_\perp^2\rho_i^2<1$ as we have applied. $I_0(\lambda_i)$ is the modified Bessel function.

5. Current density

Since the average value of current vanishes which is contributed by first-order perturbations of density and velocity due to their periodical variations, we evaluate the average current per unit wavelength which is the second-order perturbation. To evaluate the perturbed current density per unit wavelength, we use the following set of equations

$$\bar J_{i,e}=\int_0^\lambda ds\int_0^\infty 2\pi V_\perp dV_\perp\int^\infty dV_\| e$$

$$\times\left[(N+n_1)\left(\bar V+\bar u\right)-NV\right]_{i,e} \tag{17}$$

and $\bar J=\bar J_i-\bar J_e$.

With the help of eqs. (6), (9) and (11) we obtain

$$J_{xe}=-\frac{N_0e^3k_\perp k_\|^2\lambda}{2m_e^2\Omega_e^2}\left(\frac{\psi_1(\phi_1-\psi_1)}{\omega}\right.$$

$$\frac{(2\omega-4\omega_{Be})}{k_\|^2V_{T\|e}^2\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}\psi_1^2$$

$$\frac{2\omega_{Be}}{k_\|^2V_{T\|e}^2\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}$$

$$\left.\frac{4\omega^2\omega_{Be}\left[1-\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}{k_\|^2V_{T\|e}^4\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]^2}\right)\phi_1\psi_1, \tag{18}$$

$$J_{xi}=-\frac{N_0e^3k_\perp k_\|^2\lambda}{2m_i^2\Omega_i^2}\left[\phi_1\psi_1\left(\frac{1}{\omega}+\frac{\omega_{Bi}}{\omega^2}\right)\right.$$

$$\left.\frac{\omega_{Bi}}{\omega^2}\left(\frac{2T_{\|i}}{m_i}k_\|^2(\phi_1-\psi_1)\psi_1\right)\right]\left(1-k_\perp^2\rho_i^2\right). \tag{19}$$

Similarly, for the current in the z-direction

$$J_{ze}=\frac{N_0e^3\psi_1k_\|\lambda}{2m_e^2}\ \frac{1}{\Omega_e^2}\left(\frac{(\phi_1-\psi_1)}{\omega}\right.$$

$$\frac{(2\omega-\omega_{Be})\psi_1}{k_\|^2V_{T\|e}^2\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}$$

$$+\left(\frac{\omega_{Be}}{k_\|^2V_{T\|e}^2\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}\right.$$

$$\left.\frac{4\omega^2\omega_{Be}\left[1-\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}{k_\|^2V_{T\|e}^4\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]^2}\right)\phi_1\ -4(\omega-\omega_{Be})$$

$$\left.\frac{\left[1-\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}{k_\|^2V_{T\|e}^4\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}\psi_1\right) \tag{20}$$

$$J_{zi}=\frac{N_0e^3k_\|\lambda\psi_1}{2m_i^2}\left[-\frac{k_\perp^2}{\Omega_i^2}\left\{\frac{\phi_1}{\omega}+\frac{\omega_{Bi}}{\omega^2}\phi_1-\frac{k_\|^2\omega_{Bi}}{\omega^4}\right.\right.$$

$$\left.\times\left(\frac{T_{\|i}}{m_i}\right).(\phi_1-\psi_1)\right]-\frac{\psi_1}{\omega V_{T\|i}^2\left[1-\left(e^2/m_i^2\right)E_0^2/k^2V_{T\|i}^4\right]}$$

$$\left.\frac{3k_\|^2\omega_{Bi}}{\omega^3}\psi_1\right\}.\left(1-k_\perp^2\rho_i^2\right). \tag{21}$$

Substituting eqs. (19) and (18) in eq. (17), we obtain

$$
\begin{aligned}
J_{*} = \frac{ek_\perp k_\parallel^2 \lambda \psi_1}{8\pi} \frac{\omega_{pe}}{m_e \Omega_e^2} & \left\{ \frac{(\phi_1 - \psi_1)}{\omega} \right. \\
& - \frac{(2\omega - 4\omega_{Be})}{k_\parallel^2 V_{T\parallel e}^2 \left[1 + (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]} \psi_1 \\
& + \left(\frac{2\omega_{Be}}{k_\parallel^2 V_{T\parallel e}^2 \left[1 + (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]} \right. \\
& \left. + \frac{4\omega^2 \omega_{Be} \left[1 - (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]}{k_\parallel^2 V_{T\parallel e}^4 \left[1 + (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]^2} \right) \frac{\omega_{pi}^2}{m_i \Omega_i^2} \\
& \times \left\{ \phi_1 \left(\frac{1}{\omega} + \frac{\omega_{Bi}}{\omega^2} \right) - \frac{\omega_{Hi}}{\omega^4} V_{T\parallel i}^2 k_\parallel^2 (\phi_1 - \psi_1) \right\} \\
& \times \left(1 - k_\perp^2 \rho_i^2\right)
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
J_{*} = \frac{e\psi_1 k_\parallel \lambda}{8\pi} \frac{\omega_{pe}^2}{m_e} & \left\{ \frac{k_\perp^2}{\Omega_e^2} \left(- \frac{(\phi_1 - \psi_1)}{\omega} \right. \right. \\
& - \frac{(2\omega - \omega_{Be})\psi_1}{k_\parallel^2 V_{T\parallel e}^2 \left[1 + (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]} \\
& + \left(\frac{\omega_{Be}}{k_\parallel^2 V_{T\parallel e}^2 \left[1 + (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]} \right. \\
& \left. + \frac{4\omega^2 \omega_{Be} \left[1 - (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]}{k_\parallel^2 V_{T\parallel e}^4 \left[1 + (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]^2} \right) \phi_1 \\
& \left. - 4(\omega - \omega_{Be}) \frac{\left[1 - (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]}{k_\parallel^2 V_{T\parallel e}^4 \left[1 + (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]^2} \psi_1 \right) \\
& - \frac{\omega_{pi}^2}{m_i \omega} \left\{ \frac{k_\perp^2}{\Omega_i^2} - \frac{3k_\parallel^2 \omega_{Bi}}{\omega^3} \psi_1 \right. \\
& + \frac{\cdots}{V_{T\parallel i}^2 \left[1 - (e^2/m_i^2) E_0^2 / k^2 V_{T\parallel i}^4\right]} \\
& \left. \left. + \frac{k_\perp^2}{\Omega_i^2} \left(\frac{\omega_{Bi}}{\omega} \phi_1 - \frac{k_\parallel^2 \omega_{Bi}}{\omega^3} \left(\frac{T_{\parallel i}}{m_i} \right) \cdot (\phi_1 - \psi_1) \right) \right\} \right\} .
\end{aligned}
\tag{23}
$$

In the evaluation of the current densities, it was assumed that the field-aligned and perpendicular currents are due to an electromagnetic kinetic Alfvén wave and the contribution due to diamagnetic drift was neglected.

6. Energy balance and growth rate

The oscillatory motion of non-resonant electrons, carries the major part of energy [23,24]. The wave energy density per unit wavelength W_w is the sum of pure field energy and the changes in energy of the non-resonant particles $W_{i,e}$. It is observed that the wave energy is contained in the form of the oscillatory motion of the non-resonant electrons [23,24]. Thus

$$
W_{w} = W_{e} = \frac{\lambda k_\parallel^2 \psi_1^2}{16\pi} \frac{\psi_{pe}}{k_\parallel^2 (T_{\parallel e}/m_e)} . \tag{24}
$$

Now, we calculate the resonance energy W_r of the electrons per unit wavelength, that is

$$
W_r = \int_0^\lambda ds \int_0^\infty 2\pi V_\perp dV_\perp
$$

$$
\times \int_{(\omega/k_\parallel) - \Delta V_\parallel}^{(\omega/k_\parallel) + \Delta V_\parallel} \left(-\frac{1}{2} - N m_e u_z^2 + n_1 m_e u_z V_\parallel \right) dV_\parallel . \tag{25}
$$

With the help of eqs. (6), (10), (11) and (25) expanding the integrand around $V_\parallel = (\omega/k_\parallel + k_\perp \in_H V_\perp^2 / 2\Omega_e k_\parallel)$ and following the procedure as discussed in Refs. [23,24], in the limiting case of $k_\perp \rho_e << 1$ we obtain

$$
\begin{aligned}
W_r = \frac{1}{4} \lambda \psi_1^2 t \omega_{pe}^2 k_\parallel f_\parallel \left(\frac{\omega}{k_\parallel} \right) & \left[\left(1 - \frac{T_{\perp e} k_\perp \in_H \omega}{T_{\parallel ce} k_\parallel \Omega_e k_\parallel} \right) \right. \\
& \frac{\omega^2}{k_\parallel^2 V_{T\parallel ce}^2} \left(1 - \frac{k_\perp v_d^e}{\omega} \right) + \left(1 - \frac{2T_{\perp e} k_\perp \in_H \omega}{T_{\parallel ce} k_\parallel \Omega_e k_\parallel} \right) \\
& \times \frac{T_{\perp e} \omega}{T_{\parallel ce} k_\parallel} \frac{k_\perp \in_H}{k_\parallel \Omega_e} \left(1 - \frac{1}{2} \frac{k_\perp v_d^e}{\omega} \right)
\end{aligned}
\tag{26}
$$

where $ks = \bar{k} . \bar{r}$ and $\bar{k} = 2\pi/\lambda$ and $\omega_{pi,e}^2 = 4\pi N_0 e^2 / m_{i,e}$.

Using the law of conservation of energy, we calculate the growth rate of the drift kinetic Alfvén wave by

$$
\frac{d}{dt}(W_w + W_r) = 0 . \tag{27}
$$

With the help of eqs. (24) and (26), we have found the growth rate of the drift kinetic Alfvén wave as

$$
\gamma / \omega = \frac{\pi^{1/2} \omega / k_\parallel V_{T\parallel e}}{\left[1 + (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]^{1/2}} \left[\left(1 + \frac{1}{8} \frac{e^2 E_0^2}{m_e^2 k^2 V_{T\parallel e}^4} \right. \right.
$$

$$
\left. -2 \frac{\omega_{Be} \omega^2}{\omega k_\parallel^2 V_{T\parallel e}^2} \frac{1 - \frac{3}{8} (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4}{\left[1 + (e^2/m_e^2) E_0^2 / k^2 V_{T\parallel e}^4\right]} \right)
$$

$$\frac{k_\perp v_d^e}{\omega}-1\Big)+\frac{\omega_{Be}}{\omega}\Big(\frac{1}{2}\frac{k_\perp v_d^e}{\omega}-1 \quad 1+\frac{1}{8}\frac{e^2E_0^2}{m_e^2k^2V_{T\|e}^4}$$

$$\exp\,\frac{\omega^2/k_\|^2V_{T\|e}^2}{1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\Big]}\Big] \tag{28}$$

where $\omega_{He}=-\omega_{Bi}=k_\perp T_{\perp e}\,\epsilon_B/m_e|\Omega_e|$; $V_{T\perp,\|}^2=\left(2T_{\perp,\|}/m\right)$; v_d^e represents electron diamagnetic drift velocity and the value of ω for the drift kinetic Alfvén wave has to be substituted. In the case $\epsilon_B=0$ we recover the growth rate as derived by Baronia and Tiwari [17]. The kinetic Alfvén waves are generated by density inhomogeneity if the magnetic field inhomogeneity is absent. However, due to the magnetic field inhomogeneity the condition is altered.

7. Marginal instability criteria

To obtain the minimum amount of inhomogeneity required to generate the wave, we assume $\gamma/\omega=0$. Thus

$$\epsilon_B=\frac{1}{B}\frac{dB}{dy}=\frac{m_e\Omega_e}{k_\perp T_{\perp e}}\quad 1+\frac{1}{8}\frac{e^2E_0^2}{m_e^2k^2V_{T\|e}^4} \tag{29}$$

$$\text{where } F=\frac{\left[1-\frac{3}{8}\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right]}{\left[1+\left(e^2/m_e^2\right)E_0^2/k^2V_{T\|e}^4\right.}$$

$$\frac{2\omega^2}{k_\|^2V_{T\|e}^2}\quad\frac{1\;\frac{k_\perp v_d^e}{\omega}-1}{\left\{\left(k_\perp v_d^e/\omega\right)-1\right\}}\left(1+\frac{1}{8}\frac{e^2E_0^2}{m_e^2k^2V_{T\|e}^4}\right.$$

The value of electric field on which the growth rate may be substantial is given by

$$E_0\approx\;k^2 \qquad\qquad \frac{\quad}{k_\|^2V_{T\|e}}-1\;\Big)^{1/2} \tag{30}$$

The amount of inhomogeneity can be estimated for the plasma parameters mentioned in result and discussion and $\omega=4.2\times10^{-3}$ sec^{-1} and $k_\perp\rho_i=0.002$ as $\epsilon_B=4.467\times10^{-6}$ cm^{-1}. This provides the change in magnetic field at the scale of the order of 4.5 Km. Thus the wave can be generated in the plasma sheet during the onset of substorm when the thickness of the sheet becomes of the order of 4.5 Km. The estimated value of the parallel electric field is of the order of 60 mV/m which is according to the observations [19] on auroral field lines.

8. Results and discussion

In the numerical evaluation of the dispersion relation, current and growth-rate, we have used the following parameters for the auroral acceleration region [17,18,24]. However, these parameters are arbitrary and may be suitable for the auroral acceleration region. The density inhomogeneity and magnetic field inhomogeneity related by $\epsilon_B=-(\beta/2)\,\epsilon_N$ is considered.

$B_0=4300$ nT, $\Omega_i=412$ s^{-1}, $kT_{\|i}=1$ keV, $kT_{\|e}=1$ keV, $v_d^e=2000$ cm/sec, $\omega_{pi}/\Omega_i=10$.

Figures 1 and 2 show the variation of wave frequency ω (rad/sec) *versus* perpendicular wave number $k_\perp\rho_i$ for different

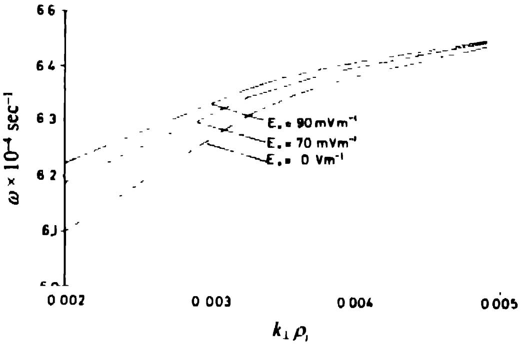

Figure 1. Frequency (ω) *versus* perpendicular wave number ($k_\perp\rho_i$) for different E_0 and $\beta=0.18$, $k_\|=2.0\times10^{-12}$ cm^{-1}

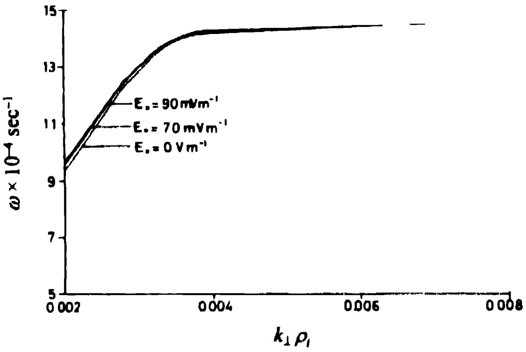

Figure 2. Same as Figure 1 with $\beta=0.46$.

values of parallel electric field (E_0) for the different plasma β. In both the figures, it is observed that parallel electric field enhances the wave frequency at the lower values of $k_\perp\rho_i$ but less effective at higher β. At higher $k_\perp\rho_i$, the effect of electric field is negligible. The higher magnetic field inhomogeneity enhance the wave frequency. The effect of both, the electric field and magnetic field inhomogeneity is to enhance the phase velocity of the wave. The increased phase velocity from the particle velocity may cause the acceleration of the charged particles by the wave particle interaction mechanism. Therefore, the parallel electric field in the inhomogeneous magnetosphere may contribute to the acceleration of charged particles through the wave.

Figures 3 and 4 predict the variation of normalized growth-rate (γ/ω) with $k_\perp\rho_i$ at different values of parallel electric field E_0 at different plasma β. It is noticed that the effect of electric field and magnetic field inhomogeneity is to reduce the growth-rate. Thus the parallel electric field controls the wave amplification in the inhomogeneous magnetosphere and transfers the energy into the particle acceleration.

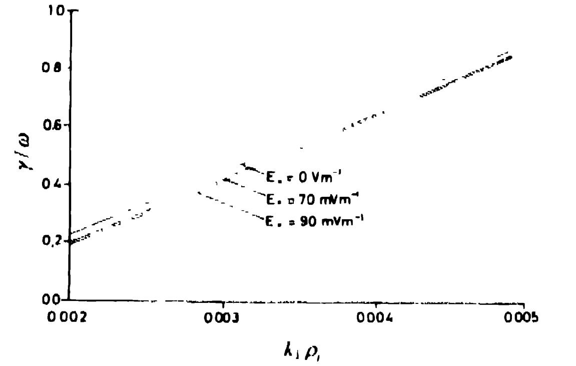

Figure 3. Growth-rate (γ/ω) *versus* perpendicular wave number ($k_\perp\rho_i$) for different E_0 and β = 0 18, $k_{\|}$ = 2 0 × 10^{-12} cm^{-1}

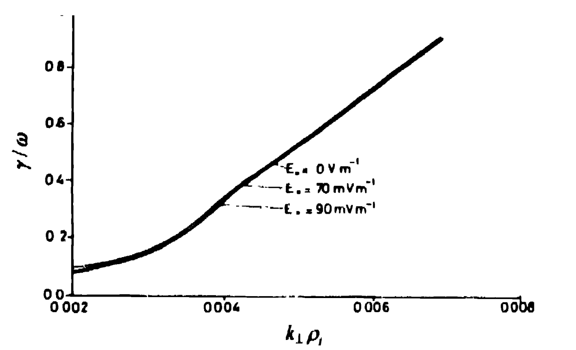

Figure 4. Same as Figure 3 with β = 0 46

Figures 5 and 6 represent the variation of field-aligned current per unit wavelength J_z with respect to $k_\perp \rho_i$ at different values of parallel electric field E_0 at different β. It can be predicted that the effect of electric field is to enhance the field-aligned currents and the major enhancement occur in the currents towards the lower $k_\perp \rho_i$ of the emission band. At the higher magnetic field inhomogeneity, the behavior of the field-aligned currents depends upon $k_\perp \rho_i$. The current decreases at lower $k_\perp \rho_i$ but increases at higher values. Thus the enhancement of the field aligned current can be predicted

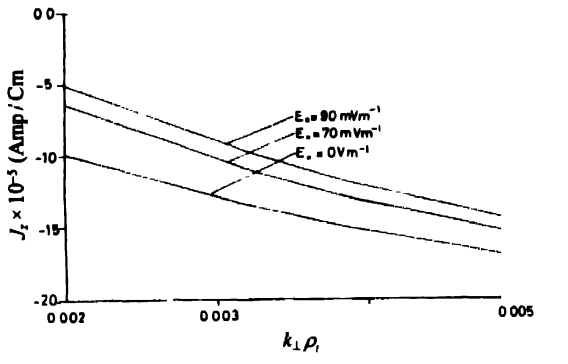

Figure 5. Parallel current per unit wavelength (J_z) *versus* perpendicular wave number ($k_\perp\rho_i$) for different E_0 and β = 0.18, $k_{\|}$ = 2.0 × 10 ^{12} cm^{-1}

in the presence of static electric field along the magnetic field in the inhomogeneous magnetoplasma in the presence of KAW which might have generated in the magnetospheric

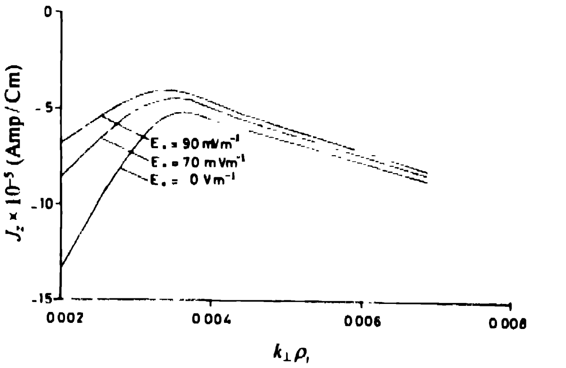

Figure 6. Same as Figure 5 with β = 0 46.

region by density inhomogeneity at the substorms times [17]. Figures 7 shows the variation of perpendicular current J_x per unit wavelength with $k_\perp \rho_i$ for different values of E_0. It is

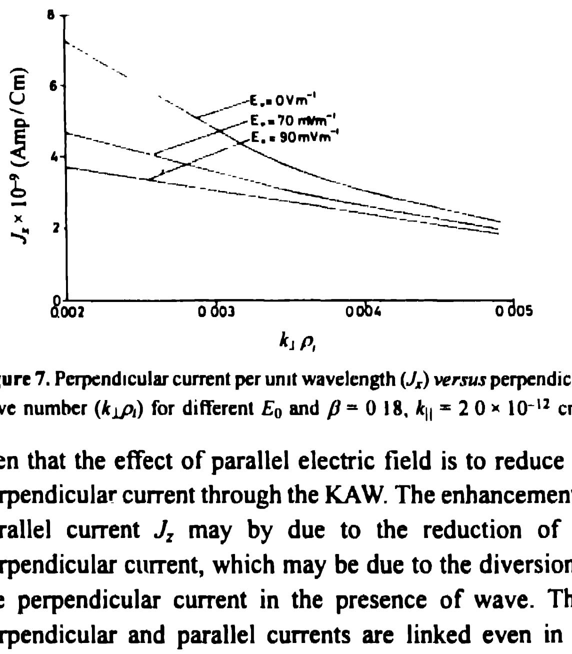

Figure 7. Perpendicular current per unit wavelength (J_x) *versus* perpendicular wave number ($k_\perp\rho_i$) for different E_0 and β = 0 18, $k_{\|}$ = 2 0 × 10^{-12} cm^{-1}

seen that the effect of parallel electric field is to reduce the perpendicular current through the KAW. The enhancement of parallel current J_z may by due to the reduction of the perpendicular current, which may be due to the diversion of the perpendicular current in the presence of wave. Thus, perpendicular and parallel currents are linked even in the presence of wave. The effect of magnetic field inhomogeneity is to reduce the perpendicular current and the effectiveness is correlated with $k_\perp \rho_i$.

Here, we may conclude that KAW can be excited by plasma density inhomogeneity as the main source of free energy [17]. Parallel electric field and magnetic field inhomogeneity can influence the growth-rate and current system once the wave has been generated by the diamagnetic drift. It follows that convection changes in the equatorial plane of the magnetosphere in a manner that produces an east-west density gradient Alfvénic disturbance will be setup that propagate to the ionosphere leading to the subauroral region II field-aligned current system and the acceleration of charged particles which are influenced by parallel electric field and magnetic field inhomogeneity [17,34]. There are variety of theories proposed for development of parallel electric field on auroral field lines [22]. Double layer process, electrostatic shock process, anomalous resistivity processes and others may be the cause of the development of parallel electric field on auroral field lines. Our aim in the present paper is to examine the effect of such a static parallel electric field on the KAW in the inhomogeneous magnetosphere.

Kinetic Alfvén wave may be generated in the distant magnetosphere and bounce between ionosphere and magnetosphere, setting a standing wave pattern. The field-aligned current and closer currents may be the effects of such a pattern which are affected by parallel electric field and magnetic field inhomogeneity. The study may be of importance in the experimental plasma also [17,35 37].

The present model is based upon Dowson's approach [38] to the Landau damping in which the wave frequency ω and wave vector $\bar{k}$ are assumed as the real. Thus, the dispersion relation contains no imaginary part and the evaluation of growth rate is done by energy exchange approach between wave and particles In this model only principal part of plasma dispersion function is used, therefore, the growth rate should not be evaluated by the dispersion relation.

Acknowledgment

We are thankful to the Indian Space Research Organisation for the financial assistance.

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